Fatigue Failure of Angle Ply Laminates

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A previously developed fatigue failure criterion for laminae in a state of plane stress is used to establish a failure criterion for first fatigue failure of balanced laminates. An extensive tension-tension fatigue testing program for *E*-glass/epoxy angle plies with various reinforcement angles has been carried out in order to test the validity of the theory. It has been found that for certain ranges of reinforcement angle there is good agreement between theory and experiment, whereas for other ranges of reinforcement angle this is not the case.

Nomenclature

E_A = lamina Young's modulus in fiber direction
E_T = lamina Young's modulus in direction transverse to fibers
f = laminate fatigue function
f_A = lamina fatigue function for load in fiber direction
f_T = lamina fatigue function for load in direction transverse to fibers
f_τ = lamina fatigue function in shear
f_f = laminate fatigue function for fiber failure mode
f_m = laminate fatigue function for matrix failure mode
G_A = lamina axial shear modulus
N = number of cycles to failure (lifetime)
n = frequency
R = ratio between minimum and maximum stress in a cycle
s = superscript denoting static failure stress
u = superscript denoting fatigue failure stress
η = nondimensional coefficient
θ = angle between fiber direction and load
μ = nondimensional coefficient
ν_A = lamina axial Poisson's ratio
σ = applied uniaxial stress
σ_A = stress in fiber direction
σ_T = stress in direction transverse to fibers
τ = shear stress

I. Introduction

THE present paper is concerned with the problem of prediction of fatigue life of laminates on the basis of known fatigue characteristics of the constituting laminae. The problem is of utmost importance to engineers designing with composite materials and has received repeated attention in the literature.[1-3]

The problem of fatigue life of single, unaxially reinforced laminae in a state of plane stress has been considered, both analytically and experimentally.[4] A fatigue failure criterion for combined plane stress was established which will be described later on. With this failure criterion, it is possible to predict first lamina fatigue failure of a laminate under cyclic loading. The major difficulty is to predict subsequent fatigue failure of laminates in the presence of failed or degraded laminae, until the whole laminate fails. The same problem arises in analysis of static failure of laminates, and it is for this reason that general failure criteria of laminates are not available at this time.

The simplest laminate is a balanced angle ply. If such a laminate is loaded symmetrically with respect to bisectors of reinforcement angles, all laminae are in the same state of stress. This state of affairs may lead to the belief that first lamina failure and laminate failure occur at the same external load. Indeed, that such is the case has been stated frequently in the literature. A recent analytical and experimental investigation[5] has shown, however, that angle ply static failure coincides with first lamina failure only for certain ranges of reinforcement angle, whereas for other angular ranges this is not the case. In the present paper a similar investigation is carried out for angle plies under cyclic loading, with the aim of assessing to what extent first lamina fatigue failure can be considered as a fatigue failure criterion of the angle ply.

II. Analytical

As was shown in Ref. 4, a uniaxially reinforced lamina in plane stress has two different failure modes, one for fiber fracture and the second for matrix fracture. The first one is expressed mathematically in the following manner:

$$\sigma_A = \sigma_A^u \tag{1}$$

with

$$\sigma_A^u = \sigma_A^s f_A(R,N,n) \tag{2}$$

The second failure mode is expressed by

$$(\sigma_T/\sigma_T^u)^2 + (\tau/\tau^u)^2 = 1 \tag{3}$$

where

$$\sigma_T^u = \sigma_T^s f_T(R,N,n) \tag{4a}$$

$$\tau^u = \tau^s f_\tau(R,N,n) \tag{4b}$$

Received June 30, 1975; revision received Sept. 8, 1975. Support by the U.S. Air Force Office of Scientific Research (AFOSR), through the European Office of Aerospace Research (EOAR), under Grant 74-2591 is acknowledged gratefully.

Index categories: Aircraft Structural Materials; Structural Composite Materials (including Coatings).

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Consider a balanced angle ply which is loaded symmetrically by cyclic uniaxial stress σ in one direction (Fig. 1). Assuming elastic laminae, the lamina longitudinal, transverse, and shear stresses are given by

$$\sigma_A = k_{xx}\sigma \tag{5a}$$

$$\sigma_T = k_{yy}\sigma \tag{5b}$$

$$\tau = k_{xy}\sigma \tag{5c}$$

where, from elastic laminate theory,

$$k_{xx} = \tfrac{1}{2}\left[1 + \sec 2\theta - \frac{(\mu + \sec 2\theta)\tan^2 2\theta}{\eta + \tan^2 2\theta}\right] \tag{6a}$$

$$k_{yy} = \tfrac{1}{2}\left[1 - \sec 2\theta + \frac{(\mu + \sec 2\theta)\ \tan^2 2\theta}{\eta + \tan^2 2\theta}\right] \tag{6b}$$

$$k_{xy} = -\tfrac{1}{2}\,\frac{(\mu + \sec 2\theta)\tan 2\theta}{\eta + \tan^2 2\theta} \tag{6c}$$

$$\mu = \frac{1 - E_A/E_T}{1 + 2\nu_A + E_A/E_T} \tag{7a}$$

$$\eta = \frac{E_A/G_A}{1 + 2\nu_A + E_A/E_T} \tag{7b}$$

Define by σ^u the cyclic external stress at which any lamina fails. If statistical scattering of failure stress is disregarded, then failure of type (1) or (3) occurs in all laminae simultaneously. Write

$$\sigma^u = \sigma^s(\theta)\ f\ (R,N,n,\theta) \tag{8}$$

Let Eqs. (5) and (8) be introduced into the failure criteria Eqs. (1) and (3) with use of Eqs. (2) and (4). Then fiber failure mode is expressed by

$$\sigma^s f_f = \sigma_A{}^s f_A / k_{xx} \tag{9}$$

and matrix failure mode is expressed by

$$(\sigma^s f_m)^2\left[\left(\frac{k_{yy}}{\sigma_T^s f_T}\right)^2 + \left(\frac{k_{xy}}{\tau^s f_\tau}\right)^2\right] = 1 \tag{10a}$$

$$f = \begin{cases} f_f \text{ for fiber failure mode} \\ f_m \text{ for matrix failure mode} \end{cases} \tag{10b}$$

For static loading the failure criteria Eqs. (9) and (10) reduce to[4,5]

$$\sigma_A^s = k_{xx}\sigma^s \tag{11a}$$

$$(\sigma^s)^2\left[(k_{yy}/\sigma_T^s)^2 + (k_{xy}/\tau^s)^2\right] = 1 \tag{11b}$$

Substitution of Eqs. (11) into Eqs. (9) and (10) yields

$$f_f(R,N,n,\theta) = f_A(R,N,n) \tag{12}$$

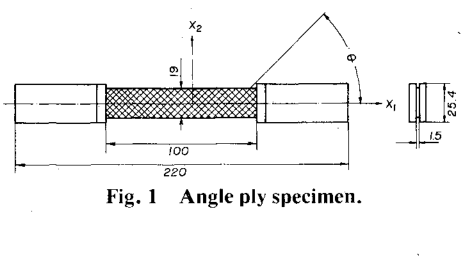

Fig. 1 Angle ply specimen.

$$f_m(R,N,n,\theta) = f_\tau\left[\frac{1 + \left(\frac{\tau^s}{\sigma_T^s}\right)^2\left(\frac{k_{yy}}{k_{xy}}\right)^2}{1 + \left(\frac{\tau^s}{\sigma_T^s}\right)^2\left(\frac{f_\tau}{f_T}\right)^2\left(\frac{k_{yy}}{k_{xy}}\right)^2}\right]^{1/2} \tag{13}$$

Equations (12) and (13) determine the laminate fatigue function in terms of lamina fatigue functions. It is seen that Eq. (12) is independent of reinforcement angle.

In order to determine which of Eqs. (12) or (13) applies, it is necessary to investigate whether failure is governed by Eqs. (9) or (10). This is done simply by use of the known lamina stresses and the fatigue failure stresses Eqs. (2) and (4), which appear in Eqs. (9) and (10), for the number of cycles at which failure is considered. It should, however, be noted that the failure criterion Eq. (9) can be operative only for very small angles (less than 2° for the present material).

The lamina fatigue functions f_A, f_T, f_τ entering into Eqs. (12) and (13) have been defined by Eqs. (2) and (4). The function f_A is determined simply by the S-N curve of a lamina under uniaxial tensile load in fiber direction. The functions f_T and f_τ may be determined by the S-N curves of an off-axis uniaxially reinforced lamina as was done in Ref. 4. It is recalled that, for the off-axis lamina, Eq. (13) is replaced by[4]

$$f''(R,N,n,\theta) = f_\tau\left[\frac{1 + \left(\frac{\tau_s}{\sigma_T^s}\right)^2 \tan^2\theta}{1 + \left(\frac{\tau_s}{\sigma_T^s}\right)^2\left(\frac{f_\tau}{f_T}\right)^2 \tan^2\theta}\right]^{1/2} \tag{14}$$

The left side of Eq. (14) is determined experimentally by the S-N curve of the off-axis lamina. If this is done for two different specimens with different θ, the functions f_τ and f_T are determined. The accuracy of Eq. (14) is tested by its fit to other angles θ. As has been shown in Ref. 4, Eq. (14) is in good agreement with experimental data.

Finally, it should be emphasized that the theory developed applies to any symmetric laminate under any cyclic membrane loading. The nature of the laminate and the loading enter into the coefficients k_{xx}, k_{yy}, and k_{xy} which appear in Eqs. (11) and (13) and are determined by laminate analysis. In general, it is necessary to apply the criterion to all different laminae in order to find out which lamina fails first.

III. Experimental

An experimental program of tension-tension fatigue on angle ply specimens made of E-glass/epoxy has been conducted in order to assess the validity of the theory just put forth.

Apparatus

Two types of fatigue machines were used for the experiments. Both are of the tension-tension type and have an eccentric cam which actuates one of the specimen grips by means of a lever. The first has a very stiff lever, so that one specimen grip attached to it is moving in a sinusoidal pattern. The other grip is attached to a very stiff load cell, thus producing constant repeated elongation amplitude. The load variation is recorded by the load cell. In laminates which exhibited deformation after some cycles, a compressive load was recorded. To avoid this, a device was attached to the load cell which kept the lower elongation at the minimum elastic strain level. This means that the amplitude difference was kept constant with continuous modification of minimum amplitude.

The second machine has a spring-type lever which connects the cam movement to the grip. Since the spring constant is low, there is almost no change in the load applied on the specimen because of permanent elongation during cycling.

Thus, this machine performs a constant load amplitude cycling.

Material and Specimen

The fatigue specimens were made of angle ply laminate. The laminate was manufactured by a filament winding technique, where a single end roving of *E*-glass fibers (Gevetex Es 13 3200X1 K921) ran through a bath of epoxy resin (Union Carbide ERL2256/ZZL0854 100:31) and was wound on a flat polygonal mandril. The polygon geometry determines the lamination angle. Eight thin, unidirectional layers were laid up alternately to produce a balanced laminate. The laminate was put into a vacuum to remove air bubbles and then was cured by heating and pressure. The laminates then were removed from the mandril and cut by a diamond wheel to the desired dimensions. Aluminum tabs were glued to the specimen for proper gripping. The specimen configuration is shown in Fig. 1. Lamination angles produced were $\pm 30°$, $\pm 35°$, $\pm 41°$, $\pm 45°$, $\pm 49°$, $\pm 55°$, and $\pm 60°$. Fiber volume fraction in laminae was 65%.

Experimental Procedure

Specimens with various lamination angles were mounted on the two fatigue machines, and their fatigue lives under different stress levels were measured. Fatigue life was measured up to 10^6 cycles. All tests were performed with amplitude ratios of $R=0.1$. (The cycling is tension-tension.) Two rates of cycling were used: $n=19$ and 1.8 cps. The tests were carried out on both machines, for constant elongation amplitude and constant load amplitude. Change of the load was recorded on a strip-chart recorder. Visual inspection of the failure mechanism was carried out for some specimens to compare with the static failure.

The material parameters of single laminae needed for the theory are elastic properties, static failure stresses, and the fatigue functions. The first were measured by strain gages and the second with an Instron machine. The lamina fatigue functions have been determined previously[4] from measurements on off-axis specimens. However, it should be noted that the epoxy used in present specimens was slightly different from the one used in Ref. 4. (The manufacturer discontinued the previously used epoxy.) The data obtained are given by

$$E_A = 5580\ \mathrm{kg/mm^2}, \qquad E_T = 1810\ \mathrm{kg/mm^2}$$

$$\nu_A = 0.285, \qquad G_A = 613\ \mathrm{kg/mm^2}$$

$$\tau^s = 8.4\ \mathrm{kg/mm^2}, \qquad \sigma_T^s = 42\ \mathrm{kg/mm^2}$$

$$f_\tau = 0.917 - 0.037 \log N \quad 10^2 < N < 10^6$$

$$f_T = 0.956 - 0.0235 \log N \quad 10^2 < N < 10^6 \tag{15}$$

The preceding results permit computation of Eq. (13) with k_{xx}, k_{yy}, and k_{xy} given by Eq. (7).

IV. Results and Discussion

Experimental fatigue failure results obtained for angle plies with $\theta = \pm 30°$, $\pm 41°$, $\pm 45°$, $\pm 49°$, $\pm 55°$, and $\pm 60°$ are shown in Figs. 2-8. In each of these figures the *S-N* curve predicted by Eq. (14) with the data of Eq. (15) is shown by a broken line. These *S-N* curves represent the occurrence of first fatigue crack in the laminae.

Additionally, $S-N$ curves have been calculated by use of static ultimate failure stress of the angle ply $\bar{\sigma}^s$ together with the fatigue function Eq. (14). This gives a "fatigue ultimate failure stress" $\bar{\sigma}^u$ defined by

$$\bar{\sigma}^u = \bar{\sigma}^s f\ (R,N,n,\theta) \tag{16}$$

The failure stress $\bar{\sigma}^s$ has been measured by use of an Instron machine. $S-N$ curves represented by Eq. (16) are shown as full lines in Figs. 2-8. It should be emphasized that Eq. (16) has no theoretical basis. Obviously, Eq. (16) predicts a higher failure stress than the one defined by first lamina failure.

It was observed that the angle plies fail in fatigue in three distinct failure modes which are similar to the ones observed in static failure:[5]

1) In angle plies with $\theta < 45°$, failure is initiated by delamination at the edges, until complete separation of the laminate occurs. The angle plies representing this group are $\theta = \pm 30°$, $\pm 35°$, and $\pm 41°$. Their $S-N$ curves are shown in Figs. 2-4. In these laminates, the fatigue life is dependent slightly on the rate of cycling, i.e., the frequency. This is shown in Fig. 2 for the $\pm 30°$ angle ply. It is seen that, in this group of laminates, the heuristic failure criterion Eq. (16) and

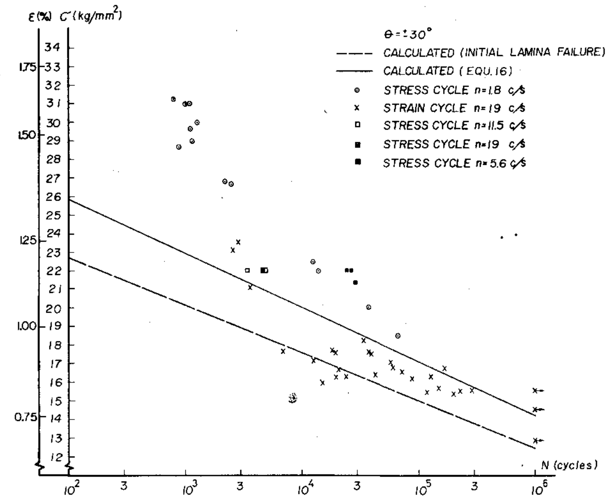

Fig. 2 Experimental and predicted $S-N$ curves of $\pm 30°$ angle ply.

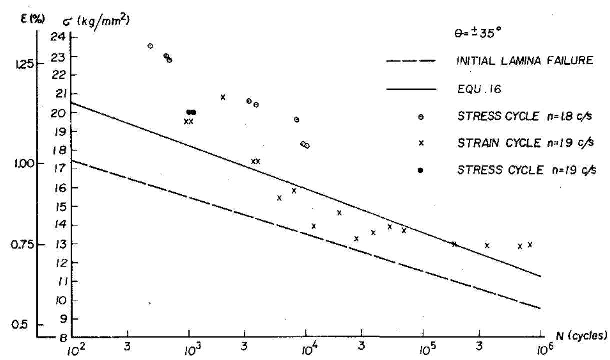

Fig. 3 Experimental and predicted $S-N$ curves of $\pm 35°$ angle ply.

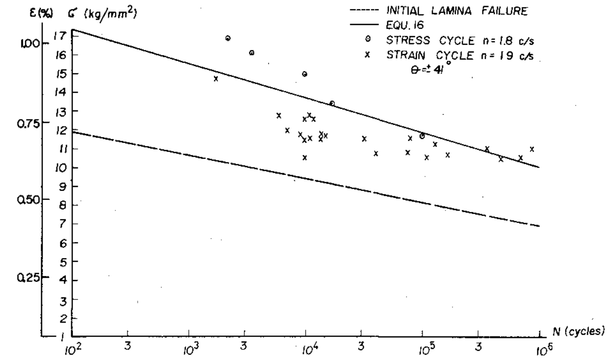

Fig. 4 Experimental and predicted $S-N$ curves of $\pm 41°$ angle ply.

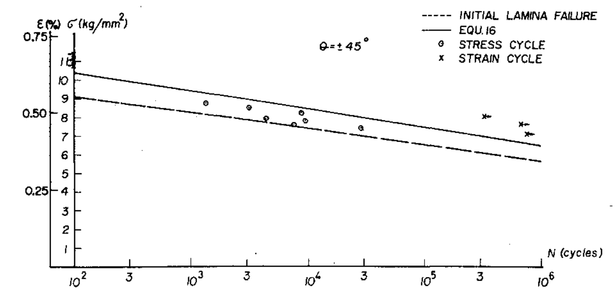

Fig. 5 Experimental and predicted $S-N$ curves of $\pm 45°$ angle ply.

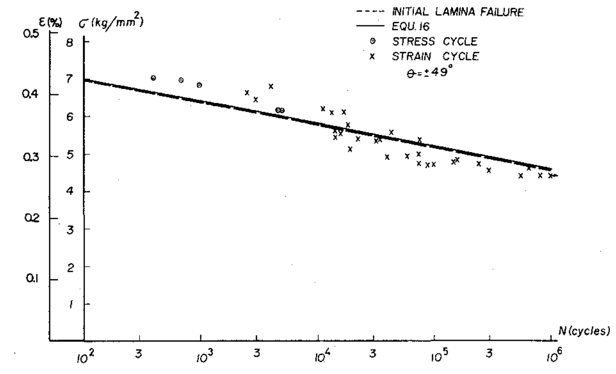

Fig. 6 Experimental and predicted $S-N$ curves of $\pm 49°$ angle ply.

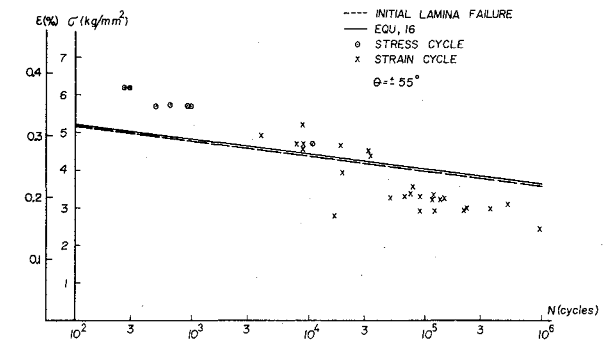

Fig. 7 Experimental and predicted $S-N$ curves of $\pm 55°$ angle ply.

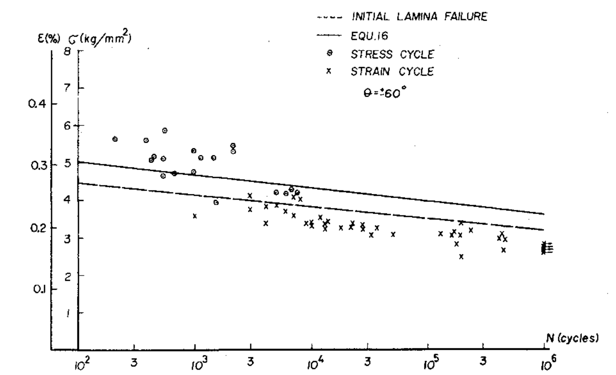

Fig. 8 Experimental and predicted $S-N$ curves of $\pm 60°$ angle ply.

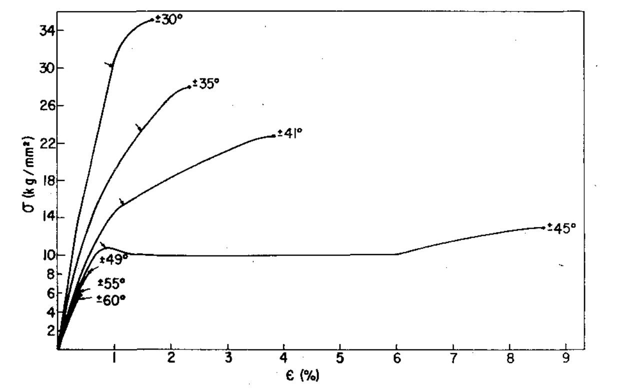

Fig. 9 Stress-strain curves of angle ply laminates.

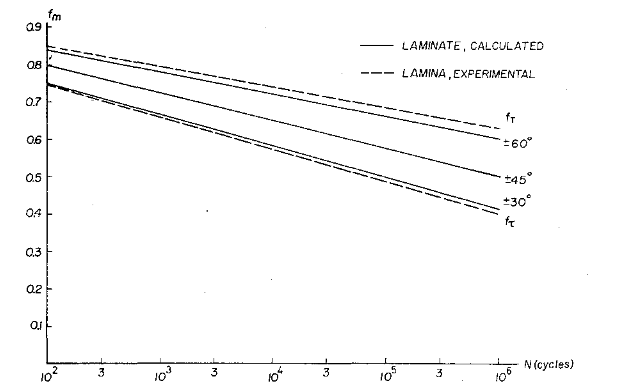

Fig. 10 Calculated fatigue functions of angle ply laminates and fatigue functions f_T and f_τ of lamina.

first lamina failure criterion predict appreciably different results. This, of course, is because static load leading to first lamina static failure and ultimate static load are appreciably different for this laminate group.[5] It is seen that the criterion Eq. (16) is distinctly in better agreement with experimental results.

2) In angle plies with $\theta \sim \pm 45°$, the failure mode consists of crack accumulation, the cracks running parallel to the fibers. Static stress-strain curve to failure of such laminates (Fig. 9) exhibits very large strains to failure and flat stress plateau. Consequently, as was observed in cycling experiments, there is essential difference between constant stress amplitude and constant strain amplitude cycling, and stiffness is reduced during cycling. It is literally impossible to fail the angle ply in constant strain cycling, since it can suffer such large strains-to-failure. On the other hand, constant stress cycling produces conventional $S-N$ curves as shown in Fig. 5. The initial stress level for the $\pm 45°$ was taken as the stress at the "knee" of the static stress-strain curve. This stress level is indicated by an arrow in Fig. 9.

3) In angle plies with $\theta > 45°$, failure occurs in sudden fashion by localized cracking. This group of laminates is represented here by $\theta = \pm 49°$, $\pm 55°$, and $\pm 60°$ shown in Figs. 6-8. There was no observed difference between constant stress cycling and constant strain cycling, nor was any influence of cycling rate found. Elastic properties changed only slightly during cycling. It may be concluded that internal damage accumulation prior to failure is not significant. Occurrence of first lamina crack literally coincides with laminate failure. It was observed previously[5] that, in this group of laminates, static first lamina failure and static ultimate failure occur literally for the same load. Consequently, first lamina fatigue failure and Eq. (16) also give the same results, as shown in Figs. 6-8 by the closeness of broken and full $S-N$ curves. It is seen that there is excellent to good agreement with experimental results.

Figure 10 shows a comparison of computed first lamina failure fatigue functions for $\theta = \pm 30°$, $\pm 45°$, and $\pm 60°$ with basic lamina fatigue functions f_τ and f_T, showing that all of the angle ply fatigue functions are bounded by f_T and f_τ. It easily is shown from Eq. (13) that, if $f_\tau < f_T$, which condition is satisfied by the present fatigue functions Eq. (15), then $f_\tau < f_m < f_T$.

Figure 11 shows comparison of computed $S-N$ curves of angle plies with measured $S-N$ curves of off-axis plies

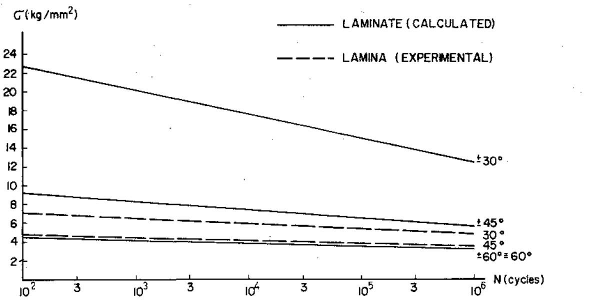

Fig. 11 Calculated $S-N$ curves of angle ply laminates and experimental $S-N$ curves of off-axis laminae.

(laminae) with the same θ. Interesting enough, the $S-N$ curves of $\pm 60°$ angle ply and the 60° off-axis specimen coincide. In order to explain this phenomenon, consider a $\pm\theta$ angle ply and off-axis specimen of angle θ, both of which are loaded by the same uniaxial load in the specimen axis direction. It can be shown by use of the results of Eq. (7) that the stresses in the laminate of the angle ply and in the off-axis specimen will be identical if and only if

$$\cos 2\theta_0 = \mu / (\eta - 1)$$

where μ and η are given in Eq. (7). In the present case it is found that $\theta_0 = 59°$.

V. Conclusion

A fatigue failure criterion which predicts first failure of laminae in a laminate has been established. Experimental investigation of fatigue failure of E-glass/epoxy laminates has shown that, for angles $\theta \geq 45°$, there is very good agreement with the given criterion. For angles $\theta < 45°$, the criterion underestimates the fatigue failure load. Better agreement with experiments is obtained by use of a heuristic criterion which uses, for fatigue failure stress, the static laminate ultimate stress multiplied by predicted first failure fatigue function.

The $\pm 45°$ angle ply is of singular nature because of its pronounced macroductile behavior in static loading. Consequently, this laminate showes a tremendous difference between constant stress cycling and constant strain cycling behavior. It cannot be failed in fatigue by the latter.

It is felt that the fatigue failure criterion may be used conservatively to assess fatigue failure of laminates on the basis of fatigue data of single laminae. It should serve to reduce necessary fatigue experimentation.

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